

# Complementary PPT tutorial to IS 8.7: Assessment of theoretical approaches to seismic network optimization

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by

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## Aim

- During the last decades the installation of seismic networks has been flourishing on local, regional and global scale.
- Efforts are made to assure best possible data products from such major investments.
- Since the 1970s theoretical modelling aimed at seismic network optimization has become fashionable.
- Yet, pretended benefits were often oversold and raised unrealistic expectations of customers.

## Therefore:

- This information sheet outlines some of the common theoretical concepts on which such optimization procedures are based as well as the assumptions that have been made to run the algorithms.
- Oversimplified assumptions or restricting conditions may not be met under real conditions.
- Moreover, optimal determination of **different** seismological parameters require **different** network configurations. Therefore, no single solution can satisfy the usually multi-purpose aim of seismic networks. ↓

- Yet, if one does not expect correct all-optimal solutions from theoretical network modelling in absolute terms but rather some guidance in a more qualitative sense for specific tasks, then such solutions may be useful.
- Two examples for specific tasks are given. For high quality location performance see also criteria outlined in IS 8.5 and 8.6. For publications, more elaborated approaches and results on network optimization see **introduction, comments, and references** in the text to this information sheet.

## **Topics of this tutorial PPT:**

1. Optimal configuration of seismic networks for location
2. Design of multi-task optimum networks for aftershock recordings
3. Network optimization with respect to main seismogenic faults
4. Detectability and EQ location accuracy modelling of seismic networks
5. Summary conclusions and recommendations for practical network optimization

# 1. Optimal configuration of seismic networks for location

**Approach A:** Hypocenter/epicenter positions are given  
⇒ search for **optimal siting of stations**

**Approach B:** Studying the **potential of a given network** with respect to variations in epicenter/hypocenter locations, noise conditions, addition or deletion of seismic stations

## 1.1 Statistical theory of optimal experimental design (Kiefer and Wolfowitz, 1960; Kijko, 1977; Silvey, 1980; Rabinovitz & Steinberg, 1990)

**The D-criterion** ⇒ **maximizing  $\det(\mathbf{A}^T\mathbf{A})$**  with  **$\mathbf{A}$**  the  **$\mathbf{N}\cdot\mathbf{p}$**  matrix of partial derivatives which are, **in the case of event location**, the observables  **$\mathbf{t}$**  (travel time) with respect to the  **$\mathbf{p}$**  components of the unknown source parameters  $\varphi$ ,  $\lambda$ ,  $\mathbf{z}$  and origin time  $\mathbf{t}_0$

⇒ **Then the D-criterion is**  $\sim \det$  of the **information matrix** of the hypocenter parameters

**Thus, a D-optimal network configuration maximizes the information matrix of the hypocenter parameters.**

**Offered D-optimal solutions are often based on too simple assumptions**  
(yet most are not required by the D-criterion itself, but rather **aim at simplifying the algorithm**)

- e.g.:**
- stations can be placed at any location (*i.e., free choice, although it depends on the task as well as ambient and logistic conditions*)
  - stations are placed at the same height on the surface (*not suitable in terrains with rough topography*)
  - validity of a standard layered **1D velocity model** above the half-space (*although lateral velocity inhomogeneities and/or anisotropy may be significant*)
  - model errors are a) uncorrelated or  
b) correlated in a known way at closely spaced stations  
(*although model inaccuracies and local anomalies are perfectly correlated at the same site and closely correlated for nearby stations*)

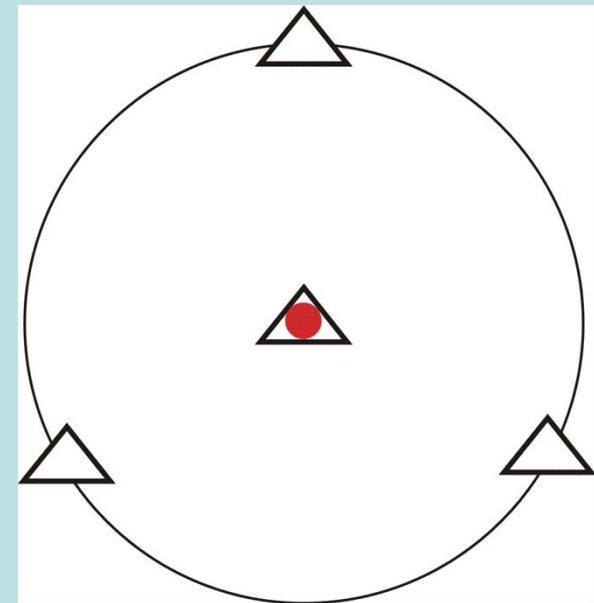
⇒ Design criterion is to maximize  $\det(\mathbf{A}^T \mathbf{W}^{-1} \mathbf{A})$  where  $\mathbf{W}$  is the **error correlation matrix**

**Warning: Such restrictive or idealized assumptions may severely limit the practical value of theoretically „optimized“ configurations**

## 1.2 Results of modelling according to the D-criterion

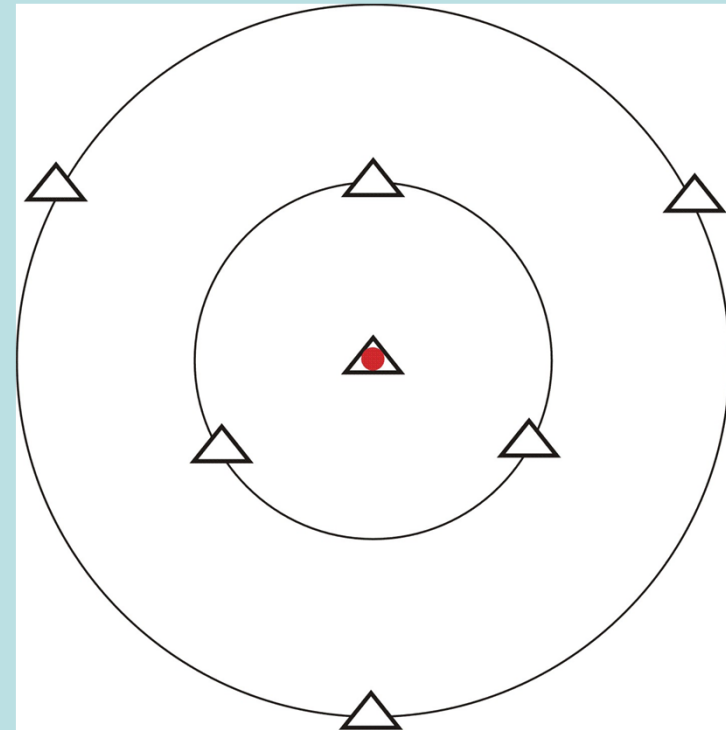
(for sources in the half-space)

- For a **given epicentre** • the **stations**  $\Delta$  of a D-optimal network are:
  - symmetric about the epicentre;
  - placed **equidistant on concentric circles** in agreement with „**seismological intuition**“.
- For a **given hypocentre in a half-space** the
  - triangular **quadripartite (4-station) network** (according to Lilwall & Francis, 1978, and Uhrhammer, 1980) **is D-optimal**;
  - The **optimal radius depends on the velocity model** since the D-criterion is proportional to the take-off angle of the seismic ray from the hypocentre to the station;
  - Therefore, the radius of the „station circle“  $r \rightarrow \infty$  for  $v = \text{const.}$



- For a **hypocentre in a layer above the half-space** (single layer crust) the D-optimal configuration consists of

- **A center station** (*although now less important than in the first case, because of...*)
- **Two concentric rings** with three equidistant stations each
- The radii of the two rings have to be chosen so that the **direct (e.g., Pg) waves** ( $\partial t/\partial z$  positive!) arrive at stations of the **inner ring** and **critically refracted waves (e.g., Pn)** ( $\partial t/\partial z$  negative!) at stations of the **outer ring**.



- Then estimates of depth  $z$  will be uncorrelated with estimates of origin time  $t_0$ .  
 ➔ **best estimates of source depth!**
- **Additional observations of S waves may substantially improve location estimates**, especially for sub-optimal configurations and for events outside of the network, **provided that** the ratio  $v_p/v_s$  is constant or a good  $v_s$  model is available (*which is, however, usually not the case because of the regrettably dominating routine practice of P-wave first motion picks only; see Gomberg et al., 1990*). **Comment 2!**

## 1.3 Conclusions

- When using only direct P waves, the high correlation between  $z$  and  $t_0$  **may** lead to an **ill-conditioned problem** and large uncertainty in the hypocenter estimate if the number of stations is insufficient and the network geometry bad.
- Networks with **mixed wave arrivals** (e.g.,  $P_g$  and  $P_n$ ) **have superior resolution**, in particular for focal depth  $z$ , since  $\partial t/\partial z$  has different signs for the upgoing direct  $P_g$  and the downgoing critically refracted  $P_n$  waves.
- This allows to design networks so that estimates of depth  $z$  will be uncorrelated with estimates of origin time  $t_0$ .
- **Additional observations of S waves may substantially improve location** under the conditions mentioned before and discussed in detail by Gomberg et al. (1990).
- Adding just **one station** to an existing N-station network may already significantly improve its performance. For more detailed discussion on this issue, however, see IS 8.7 text and **Comment 3!**

## 2. Design of multi-task optimum networks for aftershock recordings

**Approach:** **Simulated annealing** (as applied by Hardt & Scherbaum, 1994)

**Prerequisite:** List of **M possible stations locations** is available (with  $M > N$ ; N-number of selected station locations)

**Challenge:** Search for **multi-task optimal network** (e.g., for EQ location, source mechanism, tomography, assuming site specific noise levels, dynamic range, frequency band of recording systems, etc.)

**Advantages:**

- **Deployment after mainshock**, i.e., the likely **area of aftershocks is  $\pm$  known** and thus the network aperture can be chosen such that the aftershocks falls **within the network**.
- Only local distance range with  $P_g$  and  $S_g$  as first arrivals considered.
- **Prior event scenario modelling may help (?)** to select the most appropriate design for **rapid deployment** after the event and to **upgrade** the network design in response to the actual development of the aftershock distribution.

**Difficulties:**

- The fine tuning of the annealing parameters **requires experience**.
- Multi-task **optimization is not a trivial problem** and not yet satisfactorily solved.
- The normalization of **cost functions**, their **weighting**, the high **non-linearity** of systems with more than 10 stations ( $\rightarrow$  see **Comment 4!**)



## 2.1 Method of simulated annealing (SA)

- SA is a discrete inversion technique such as genetic algorithms.
- SA is fast, easy to program and not restricted to the linearity of the problem.
- SA allows **optimizing** of seismic networks for **difficult tasks**.
- SA originates from statistical mechanics and **describes the way a liquid freezes** and forms crystals; if cooling is slow enough, nature **finds a minimum energy state**
- In seismological application **ENERGY** is replaced by the **OBJECTIVE FUNCTION (OF)** which describes the **PERFORMANCE** of a seismic network.
- Thus, the process of **SA can be applied to** the problem of **finding an optimum network configuration: a) for a given task; b) for a combination of tasks.**
- SA starts from **a randomly chosen network configuration selected out of a list of  $N_p$  possible sites for  $N_s$  stations with  $N_p \gg N_s$ .**  
(However, the  $N_p$  possible sites are usually not known before the deployment of a net but assumed, e.g., to form a dense regular grid, as in the following demonstration!)
- For this **OF is computed**, i.e., the 'energy state' of the **starting configuration  $E_0$ .**
- Then **step by step new configurations** are produced by moving randomly one **station** to another site.
- **New configurations** are accepted if  $dE_i = E_i - E_0 < 0$ , **until  $E_{\min} \ll E_0$  is found.**  
 $N_{os}$  is then the number of optimally distributed station sites
- SA accepts also **conditions** such as  $E(i) > E(i-1) \rightarrow$  **avoid getting trapped in local minima**  $\rightarrow$  see **Comment 5!**

## 2.2 SA optimal networks for hypocenter location of events **within** network

(Plot examples from Hardt and Scherbaum (1994) were color-coded for eased recognition)

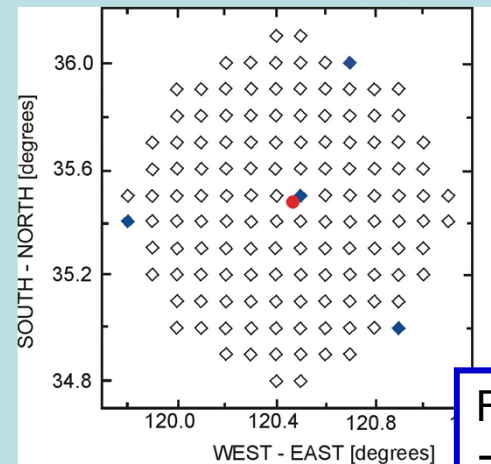
**Red dots:** number  $N_e$  of assumed event epicenters; **open diamonds:** number  $N_p$  of assumed possible station sites; **blue diamonds:** number  $N_{os}$  of calculated optimal station sites;  $N_s$  = number of available station sites.

**Single event** 4 Stations

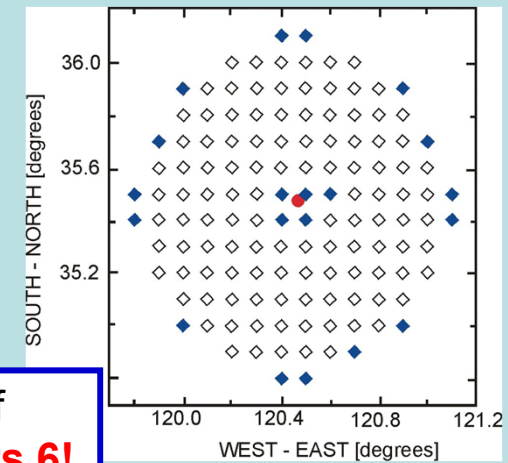
- Results as for D-criterion

→ **quadripartite network**

(forms an equal-sided triangle with one station in the centre)

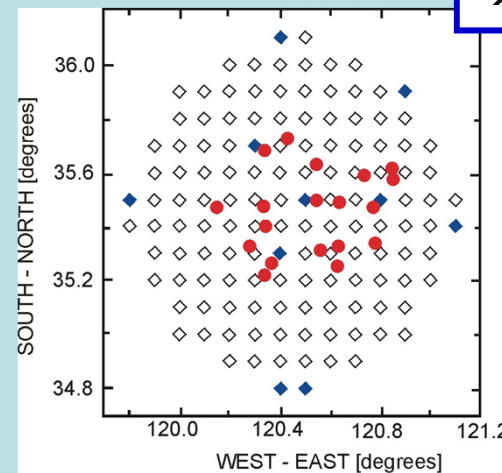


20 Stations

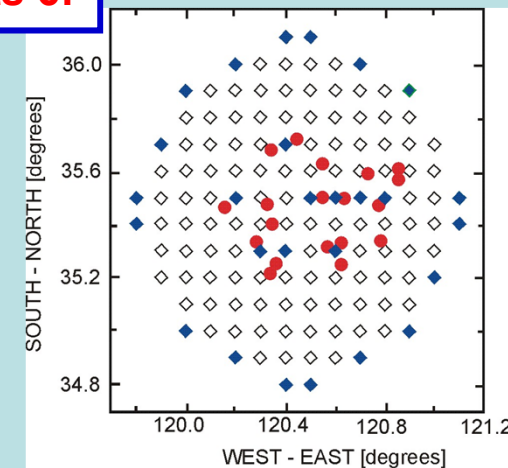


For discussion of  
→ see **Comments 6!**

**20 events** 10 Stations



30 Stations

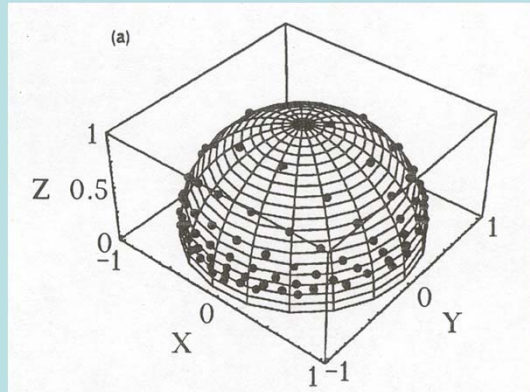


**Conclusions:** a) If event number  $N_e \ll N_s \Rightarrow$  highly redundant optimal sites; b) Only networks for  $N_e \gg N_s$  are of interest; c) Yet: The distribution of the  $N_{os}$  optimal station sites depends on the event distribution within the network!

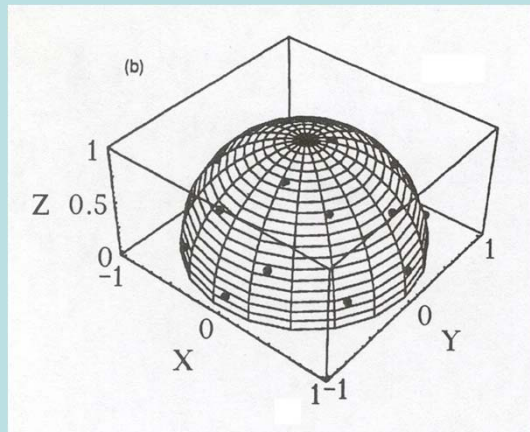
## 2.3 SA optimal network for determination of source mechanism

(Plot examples from Hardt and Scherbaum (1994) were color-coded for eased recognition)

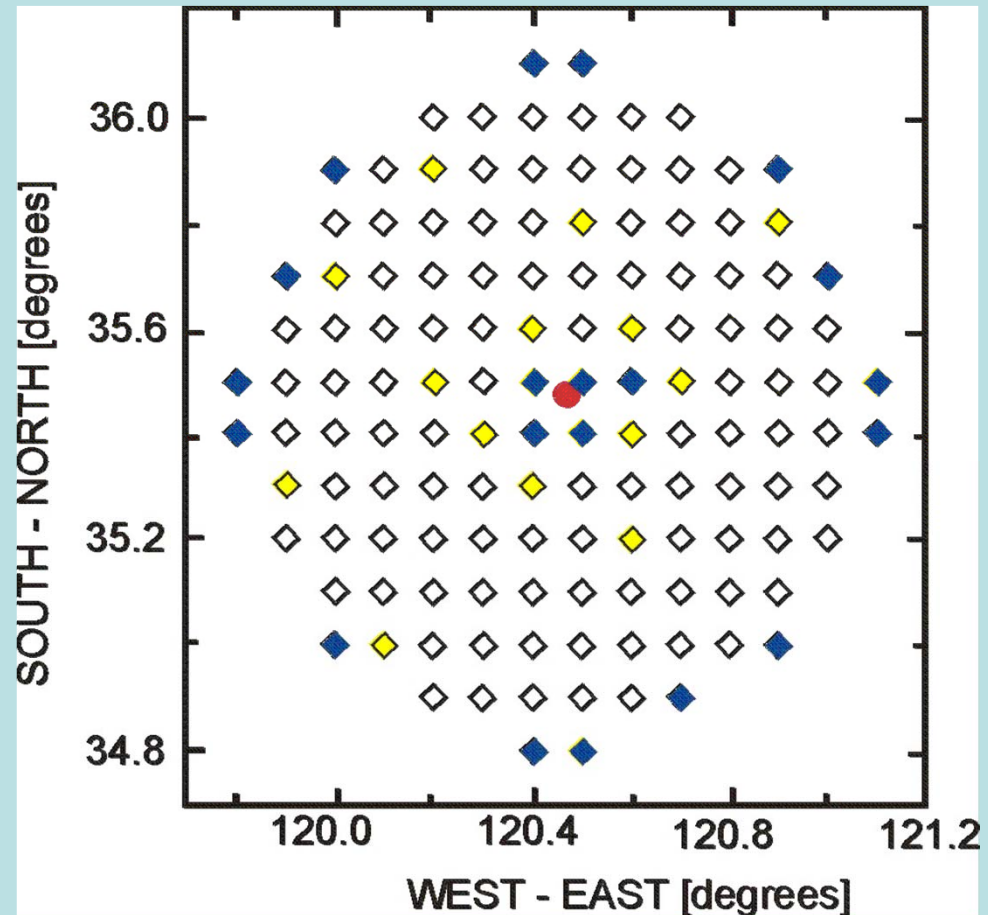
- Ray penetration points on the focal hemisphere of a **single event** in the network centre to:



all 132 assumed possible stations;



20 optimum stations, which record seismic rays that homogeneously sampled the focal sphere.



Optimum 20 station network for **source mechanism** determination of a single event in the network center.

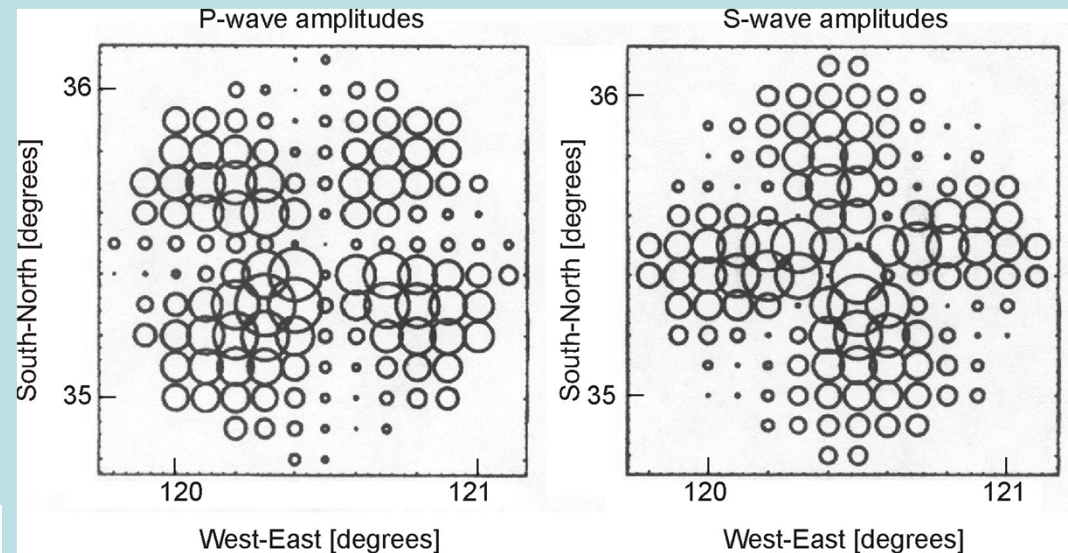
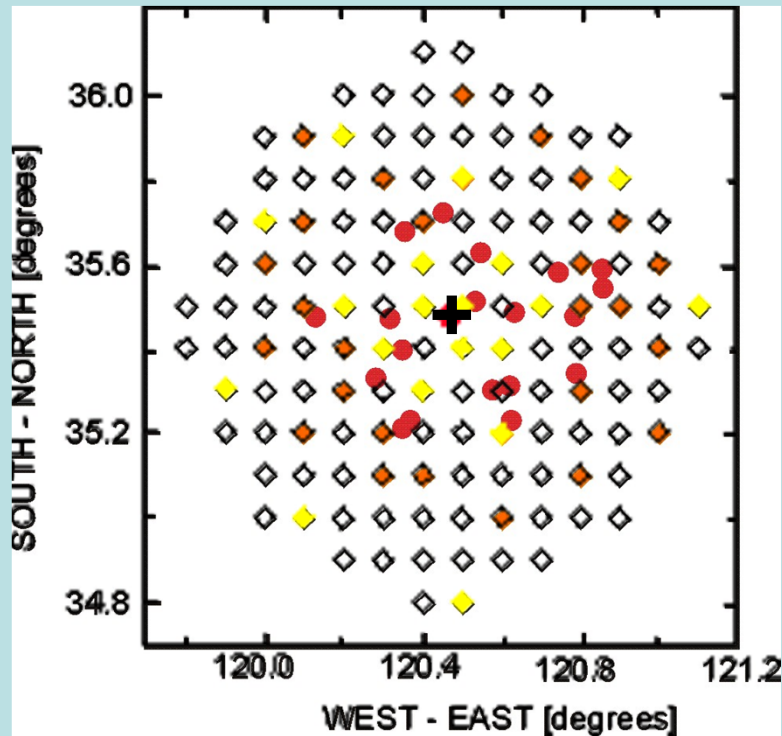
**Overlay:** Optimum 20 station network for **location** of this event

**Conclusion: Optimal networks for location are not optimal for source mechanism!**



## However:

- P and S waves have different radiation pattern
- Amplitude distribution within the network varies also with the
  - spatial orientation of rupture
  - source kinematics/directivity
  - source depth and velocity structure



Azimuthal **amplitude radiation pattern** for P- and S-waves from a pure vertically dipping and/or N-S/E-W trending **strike-slip fault**.

**Note:** The pattern varies with type of **source mechanism** and the specifics of rupture kinematics (slip direction, rupture speed → **source directivity**).

## Moreover:

- Most events occur outside of the network center!

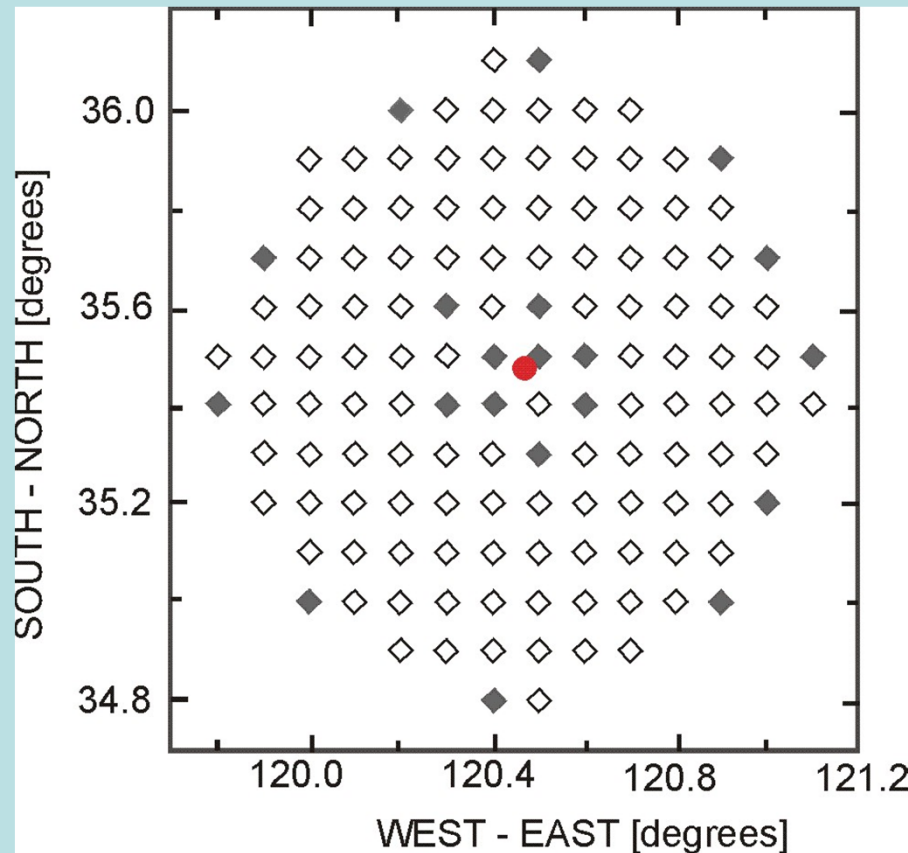
Optimum **30 station network** for source mechanism

determination of **20 distributed events** ●

Overlay: Optimum **20 station network** for source mechanism  
of one EQ **+** in network center

## 2.4 Combined SA optimal network for location and source mechanism

(Plot example taken from Hardt and Scherbaum, 1994; color-coded for eased recognition)



Optimum **20 station network** for both event location and source mechanism of a **single event** • in the network center.

- This distribution is similar to a location optimal network for a single event in the network center.

**However, the solution depends on:**

- the weighting factor given to hypocenter location and source mechanism reliability, respectively;
- the velocity structure (especially with respect to the clustering near the network center for improved source mechanism;

**and**

- optimizing for many events distributed within the net changes again the configuration of “optimal sites”.

## 2.5 SA optimal network configuration for tomography (3-D velocity and attenuation structure)

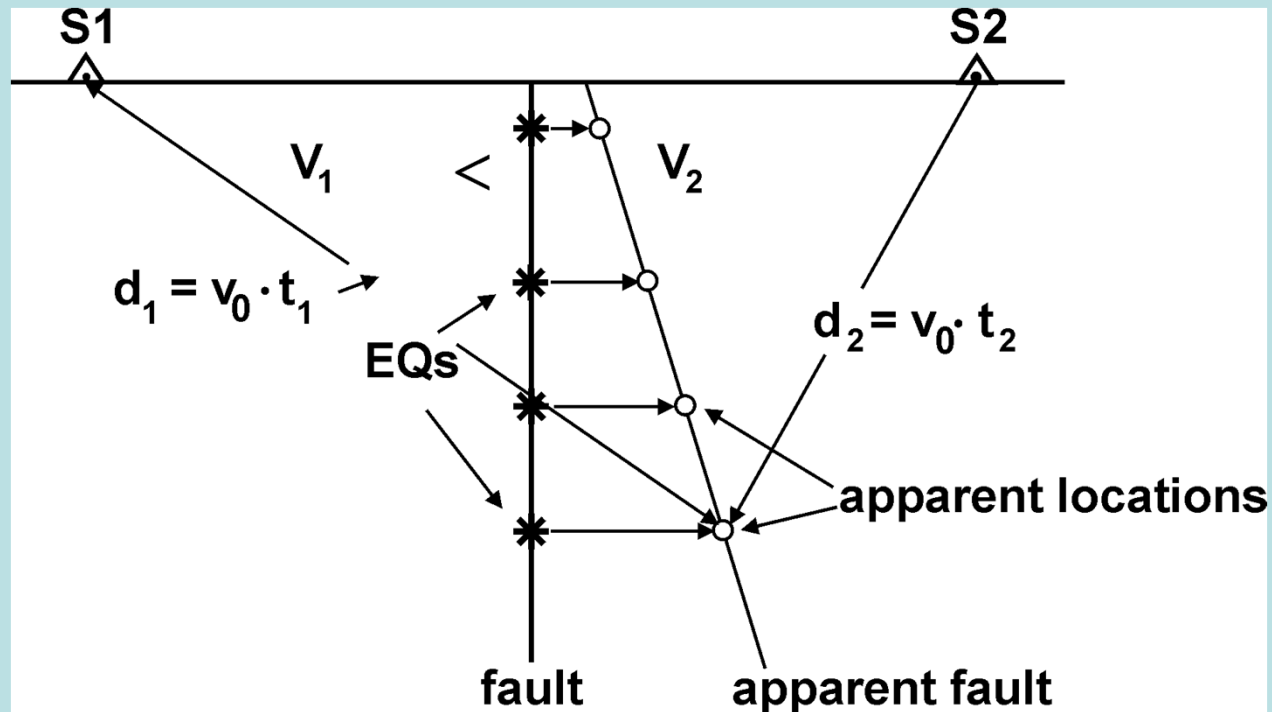
- Quality measure for tomography is the **model resolution matrix MRM** (Menke, 1989)
- Optimum resolution is obtained if the **MRM is diagonal**
- As **OF** one uses a **measure of diagonality of the MRM**, scaled between
  - 0 – for poor resolution
  - 1 – for optimum resolution
- The calculation of the MRM is rather time consuming
- **For an optimal net of 30 stations for 20 events**  
Hardt & Scherbaum (1994) calculated that this network would have  
**only < 30% optimality for tomography!**

## 2.6 Conclusions from applying SA for network optimization

- **Optimized networks for LOCATION  $\neq$  SOURCE MECHANISM  $\neq$  TOMOGRAPHY  $\neq$  ???**
- Hardly one knows prior to an EQ the list of **potential and accessible** sites in its future aftershock area. The location and size of this area strongly depends on the unpredictable accurate location and size of the future rupture.
- **In reality** the number of deployed stations  **$N_s \ll N_e$**  (number of aftershocks) and in the wake of a disaster one can hardly access predetermined “ideal locations”.
- **Aftershocks are not at all randomly distributed** within a network (but see also qualifying **Comment 7**) yet will often occur :
  - near the borders or even outside of it;
  - within an  $\pm$  elongated source volume, which is **closely related to the main rupture** and its orientation in space, and to adjacent fault systems activated due to stress redistribution in the surroundings of the main rupture.
- **Therefore**, it is more promising to **optimize location after a tomographic study** of the 3D velocity inhomogeneities in the aftershock area has been made.

## Principle of mislocation due to lateral velocity inhomogeneities

(Figure 13 of IS 11.1)



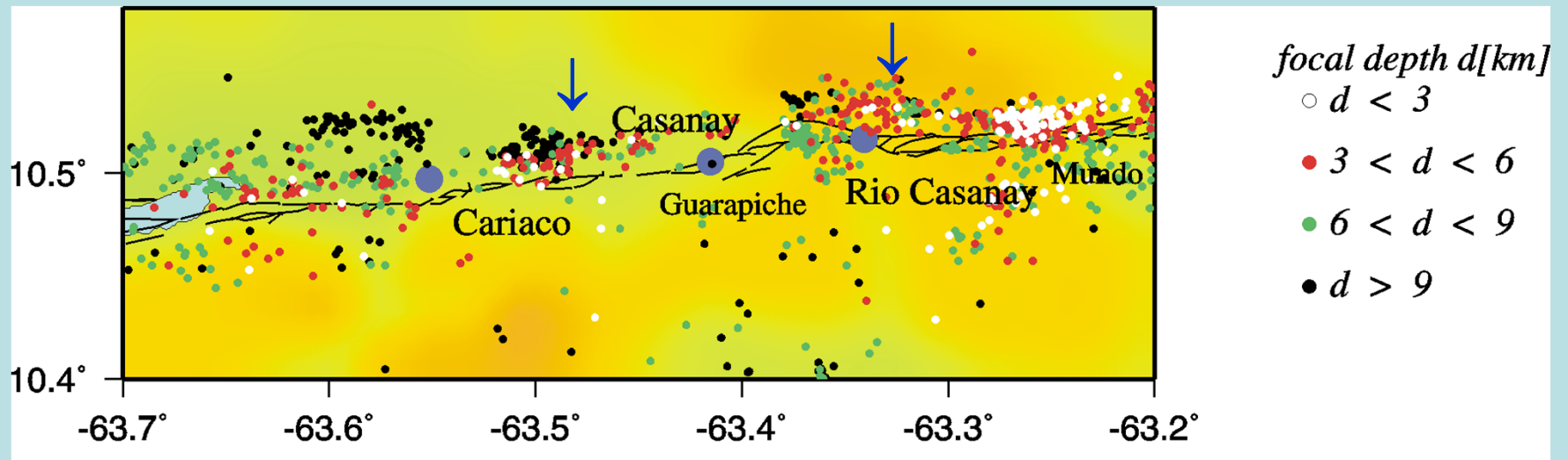
**Cartoon** illustrating systematic mislocation of earthquakes along a fault with strong lateral velocity contrast.  $v_0$  is the assumed model velocity with  $v_2 > v_0 > v_1$ .

**Note:** Events located in an inhomogeneous medium by using a laterally homogeneous velocity model are always shifted into the direction of higher velocities, the more the less homogeneous the azimuthal distribution of stations used in the location procedure.

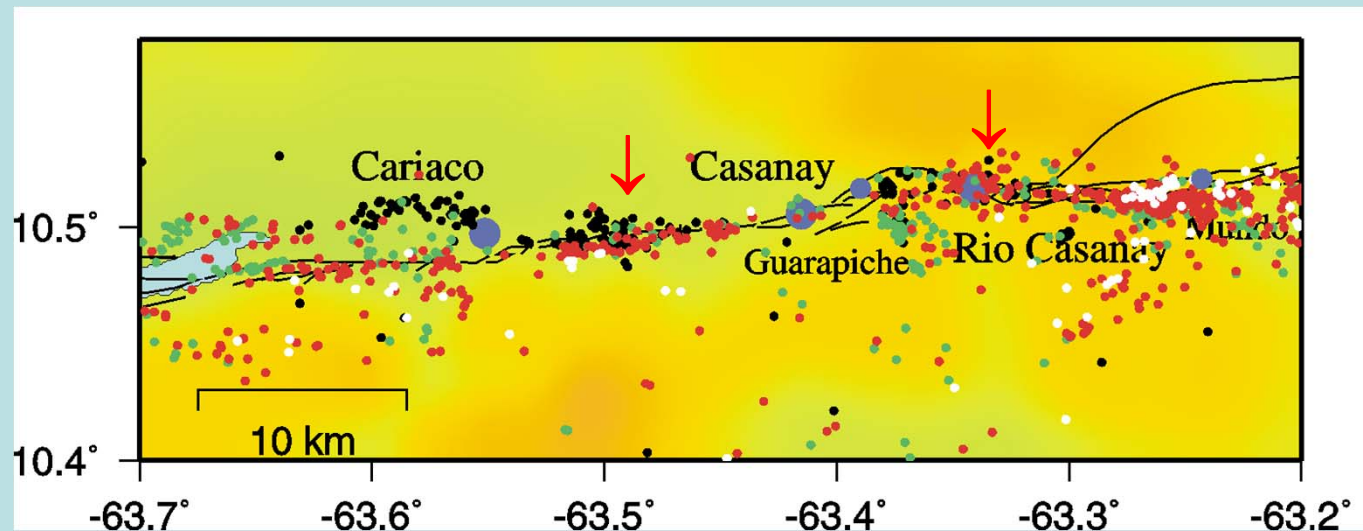


## REAL CASE STUDY

### Epicenter maps of aftershocks of the 1997 Ms6.8 Cariaco earthquake, NE Venezuela.

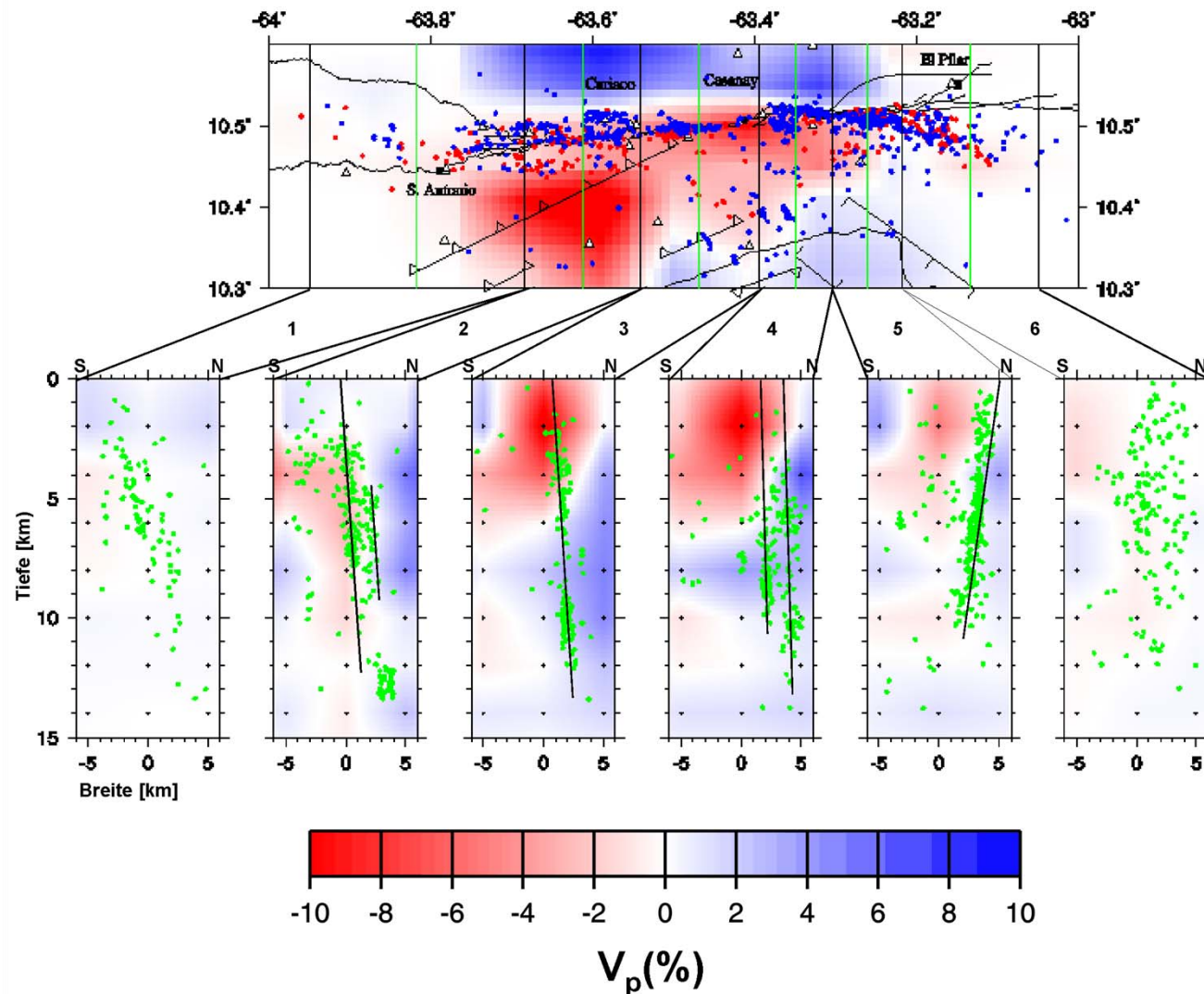


**Top:** Results from HYPO71 locations based on a 1-D velocity-depth model.



Note the shift  
of epicenters  
from North ↓  
before  
towards South ↓  
after  
3-D relocation

**Bottom:** Relocation of the aftershocks on the basis of a 3-D model derived from a **tomographic study** ⇨ of the aftershock region (Figure 11 of IS 11.1, courtesy of M. Baumbach, H. Grosser and A. Rietbrock, 2001).



**Note** the significantly higher P-wave velocities towards North !

**Note** the excellent linear alignment of most aftershocks with depth along the Cariaco fault system in the center area of good tomographic control and resolution and the wide scatter of hypocenter locations outside this area of good network coverage. (where deviations of velocity from the average model can not be resolved).

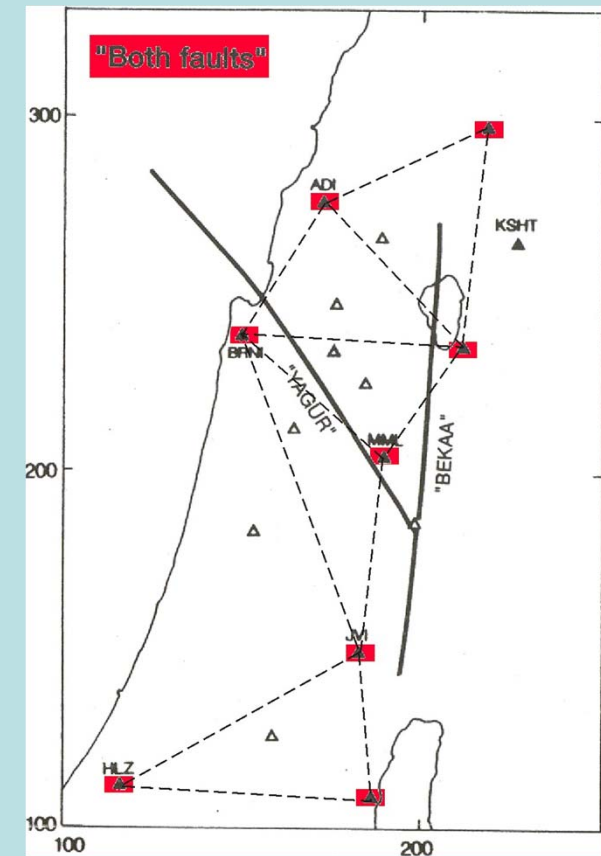
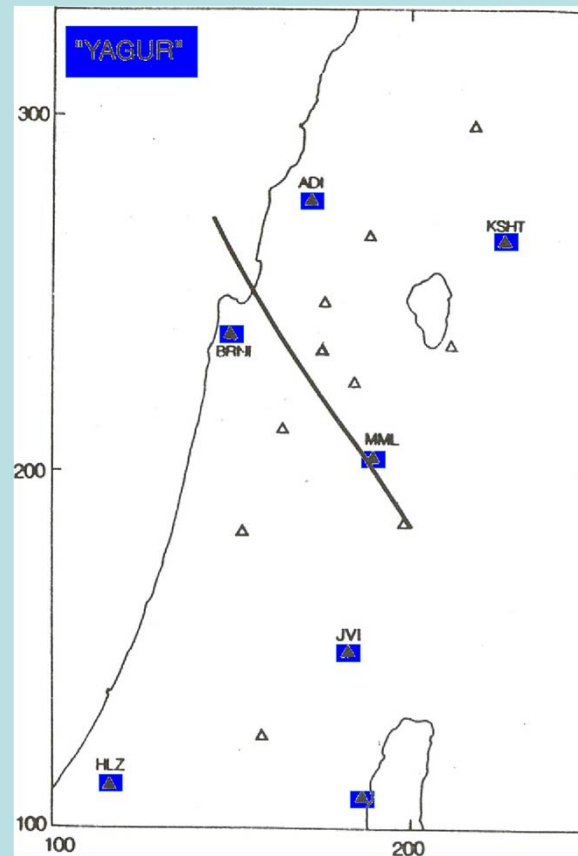
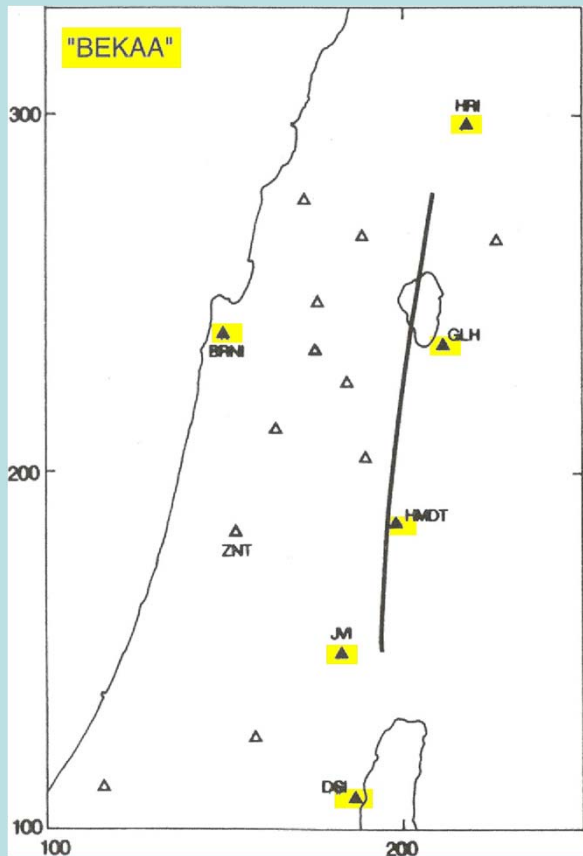
**3-D distribution of the P-wave velocity** in the focal region of the 1997 Cariaco earthquake as derived from a tomographic study. The map view shows the velocity distribution in the layer between 2 km and 4 km depth. Red and blue dots mark the epicenters of the aftershocks. The red ones were chosen because of their suitability for the tomography.

**Vertical cross sections** showing the depth distribution of the aftershocks (green dots) together with the deviations of the P-wave velocity from the average reference model. The depth range and the lateral changes of fault dip are obvious (Figure 12 of IS 11.1, courtesy of M. Baumbach, H. Grosser and A. Rietbrock, 2001).

### 3. Network optimization with respect to main seismogenic faults

(Case study by Steinberg et al., 1995)

- Tasks:**
- Reduction of an existing network from 17 stations to 6-8 station sites
  - Optimal 6-8 stations for monitoring the two main faults “Bekaa” and “Yagur”



Optimal six-station network (out of 17 existing stations) for monitoring the “Bekaa” fault (yellow) and “Yagur” fault (blue), as well as of an optimal eight-station network monitoring both fault systems (red) (according to Steinberg et al., 1995)

## 4. Detectability and EQ location accuracy modeling of seismic networks

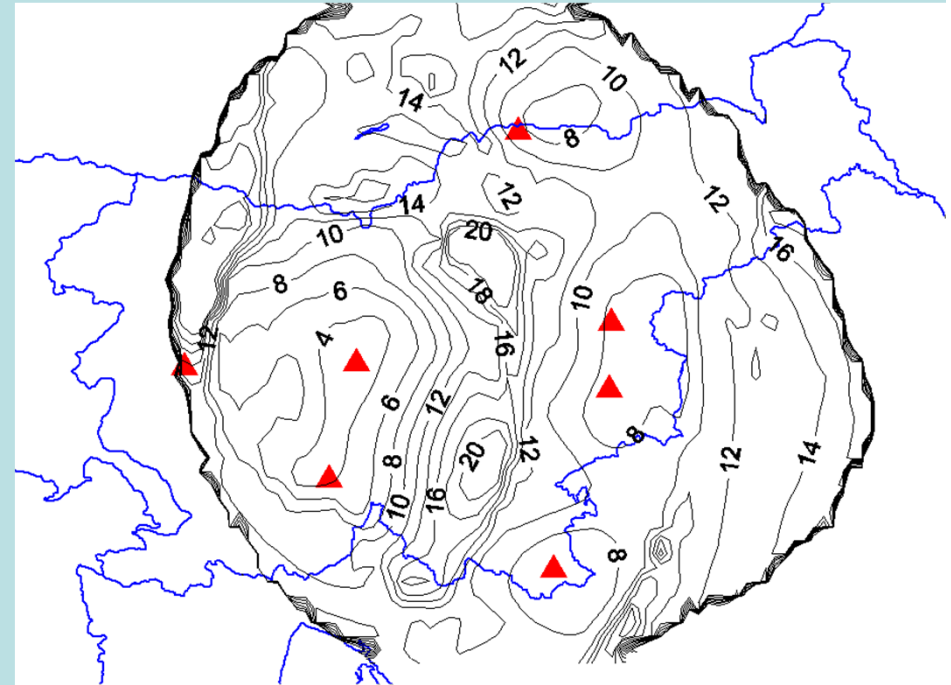
according to Živčić and Ravnik (see IS 7.4)

### Allows to calculate accuracy of event location:

- within and nearby outside of network;
- for different hypocenter depth and EQ magnitudes (i.e.,  $\neq$  SNR and accuracy of time picking; see also **Comment 8**).
- to recalculate network performance when configuration and/or number of station is changed.

### Assumptions:

- Station coordinates are exactly known;
- RMS value of noise in the used frequency range is known;
- instrument response flat between 1 - 10 Hz;
- P- and S- arrival times picked with known accuracy;
- P and S velocity model known (with some uncertainty);
- small local network which allows to use the flat Earth approximation.



Result of model calculations for the Stareslo network in Slovenia for  $M_l = 1.0$  earthquake.

Red triangles: station positions

Blue lines: borders of Slovenia,

Thin black lines: isolines of **hypocenter location error in km**;

Thick black line outer boundary: outer limit of the network's location capability for earthquakes of  $M_l = 1.0$ .

The computer program LOK by Živčić and Ravnik is described in more detail in IS 7.4 and can be downloaded from there.



## 5. Summary conclusions and recommendations for network optimization

1. **Theoretically optimal network geometries** obtained under idealized assumptions or restricting conditions **may not be optimal under real conditions** of
  - spatially distributed,  $\pm$  irregular seismicity patterns;
  - significant variations in velocity structure;
  - strongly varying SNR conditions even at “optimal sites” (see **Comment 8!**);
  - unaccessibility or other limitations of “optimal sites” due to lacking infrastructure and power supply, rough topography, bad site geology, data-link problems etc.
2. Station distributions should provide a **good azimuthal coverage** with gaps  $\leq 120^\circ$  and **nearly equidistant** spacing forming  $\pm$  equal-sided station triangles.
3. **Good depth estimates require** at least **one station near epicentre** (at  $D < 2$  h), or **stations both in the Pg and Pn distance range, or well identified depth phases.**

**THEREFORE: priority should be given to:**

- **Meeting conditions 2. and 3.** together with comparably **good detectability thresholds** (SNR) at the various stations/subnets as far as possible;
- **later object and/or problem-oriented optimization of existing networks** by addition or deletion of stations based on better knowledge of **seismicity patterns, velocity inhomogeneities, SNR, chief aim of study, financing** etc. (**see Final Comments**).